A Universal Spectrum for Interannual Variability of Monsoon Rainfall over India

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ABSTRACT

Continuous periodogram analyses of 115 years (1871–1985) summer monsoon rainfall over the Indian region show that the power spectra follow the universal and unique inverse power law form of the statistical normal distribution with the percentage contribution to total variance representing the eddy probability corresponding to the normalized standard deviation equal to $[(\log L / \log T_{50}) - 1]$ where L is the period length in years and T_{50} the period up to which the cumulative percentage contribution to total variance is equal to 50. The above results are consistent with a recently developed non–deterministic cell dynamical model for atmospheric flows. The implications of the above result for prediction of interannual variability of rainfall is discussed.

I. INTRODUCTION

The interannual variabilities of atmospheric flows as recorded in meteorological parameters such as windspeed, temperature and pressure at the earth's surface and in the atmospheric column extending up to the stratosphere have been investigated extensively and major quasiperiodic oscillations such as the QBO (quasi–biennial oscillation) and the 2–5 year ENSO (El Nino / Southern Oscillation) cycle have been identified (Lamb, 1972; Philander, 1990). Such dominant cycles are however superimposed on an appreciable "background noise" contributed by a continuum of eddies of all scales within the time, space scales investigated (Lorenz, 1990; Tsonis and Elsner, 1990). It is important therefore, to identify the physics of multiple scale interactions (Barnett, 1991) and quantify the total pattern of fluctuations of atmospheric flows for predictability studies. Long–range spatiotemporal correlations manifested as the selfsimilar fractal geometry to the global cloud cover pattern and the inverse power law form for atmospheric eddy energy spectrum documented by Lovejoy and Schertzer (1986) are signatures of self–organized criticality (Bak et al., 1988) or deterministic chaos (Mary Selvam, 1990; Mary Selvam et al., 1992) in atmospheric flows. The physics of self–organized criticality is not yet identified. In this paper a cell dynamical model for atmospheric flows (Mary Selvam, 1990) is summarized. The model predicts self–organized criticality as intrinsic to quantum–like mechanics governing atmospheric flows and, as a natural consequence leads to the result that the atmospheric eddy energy spectrum represents the statistical normal distribution. The model predictions are in agreement with continuous periodogram analyses of 115 years summer monsoon rainfall over the Indian region. Such unique quantification, namely the inverse power law form of the statistical normal distribution for the atmospheric eddy energy spectrum implies predictability of the total pattern of atmospheric fluctuations. The applications of the above result for prediction of interannual variability of atmospheric flows are discussed.

II. CELL DYNAMICAL SYSTEM MODEL

In summary, (Mary Selvam, 1990; Mary Selvam, et al., 1992) the mean flow at the planetary atmospheric boundary layer (ABL) possesses an inherent upward momentum flux of surface frictional origin. This upward momentum flux is progressively amplified by the exponential decrease of atmospheric density with height coupled with latent heat released during microscale fractional condensation by deliquescence on hygroscopic nuclei even in an unsaturated environment. This mean upward momentum flux generates helical vortex roll (or large eddy) circulations in the ABL seen as cloud rows / streets, mesoscale cloud clusters (MCC) in the global cloud cover pattern. Townsend (1956) has shown that large eddy circulations form as the spatial integration of enclosed turbulent eddies intrinsic to any turbulent shear flow (Fig. 1). The relationship between the root mean square (r.m.s.) circulation speeds W and w_* of large and turbulent eddies of respective radii R and r is then obtained as:

$$W^2 = \frac{2r}{\pi R} w_*^2 . \tag{1}$$

A continuum of progressively larger eddies grow from the turbulence scale at the planetary surface with two-way ordered energy feedback between the larger and smaller scales as given in Eq. (1). Large eddy is visualized as the envelope of enclosed turbulent eddies and large eddy growth occurs in unit length step increments equal to the turbulent eddy fluctuation length r. Such a concept is analogous to the non-deterministic cellular automata computational technique where cell dynamical system growth occurs in unit length step increments during unit intervals of time (Oona and Puri, 1988). Also, the concept of large eddy growth in length step increments is equal to r, the turbulence length scale, i.e. length scale doubling is identified as the universal period doubling route to chaos eddy growth process. The large eddy of radius

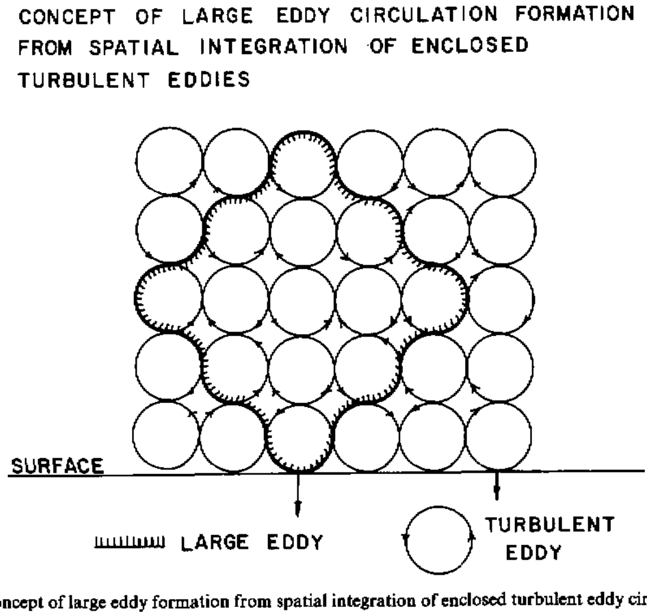

Fig. 1. Concept of large eddy formation from spatial integration of enclosed turbulent eddy circulation.

R_n at the nth stage of growth goes to form the internal circulation for the next stage, i.e. $(n+1)$th stage of large eddy growth. Such a concept, leads as a natural consequence, to the result that the successive values of the radii R and the r.m.s. eddy circulation speeds W follow the Fibonnaci mathematical number series, i.e. the ratio of the successive values of R (or W) is equal to Γ, the golden mean $[=(1+\sqrt{5})/2=1.618]$.

The overall envelope of the large eddy traces a logarithmic spiral with the quasi-periodic Penrose tiling pattern for the internal structure. Atmospheric circulation structure therefore consists of a nested continuum of vortex roll circulation (vortices within vortices) with a two-way ordered energy flow between the larger and smaller scales. Such a concept is in agreement with the observed long-range spatiotemporal correlations in atmospheric flow patterns.

The cell dynamical system model also predicts the following logarithmic wind profile relationship in the ABL.

$$W=(w_* / k)\ln Z, \tag{2}$$

where the Von Karman's constant k is identified as the universal constant for deterministic chaos and represents the steady state fractional volume dilution of large eddy by turbulent eddy fluctuations. The value of k is shown to be equal to $1/\Gamma^2(=0.382)$ where Γ is the golden mean. The model predicted value of k is in agreement with observed values. Since the successive values of the eddy radii follow the Fibonacci mathematical number series the length scale ratio Z for the nth step of eddy growth is equal to $Z_n = R_n / r = \Gamma^n$. Further, W represents the standard deviaton of eddy fluctuations, since W is computed as the instantaneous r.m.s. eddy perturbation amplitude with reference to the earlier step of eddy growth. For two successive stages of eddy growth starting from primary perturbation w_*, the ratio of the standard deviations W_{n+1} and W_n is given from Eq. (2) as $(n+1)/n$.

Denoting by σ, the standard deviation of eddy fluctuations at the reference level $(n=1)$, the standard deviations of eddy fluctuations for successive stages of eddy growth are given as integer multiples of σ, i.e. σ, 2σ, 3σ etc.

The concept of large eddy formation as the spatial integration of enclosed turbulent eddies leads as a natural consequence to the result that the atmospheric eddy energy spectrum follows normal distribution characteristics, i.e. the square of eddy amplitude represents the eddy probability density. Incidentally, the above result, namely that the additive amplitudes of eddies when squared represent the eddy probability density is inherent to the observed sub-atomic dynamics of quantum systems and is accepted as an *ad hoc* assumption in quantum mechanics (Maddox, 1988).

Atmospheric flow structure therefore follows quantum-like mechanical laws where the eddy energy spectrum represents the eddy probability density and the apparent wave-particle duality is physically consistent in the context of atmospheric flows since the bimodal (formation and dissipation) form for energy manifestation in the bidirectional energy flow intrinsic to eddy circulations results in the formation of clouds in updrafts and dissipation of clouds in downdrafts. The physical concept of quantum-like mechanics in atmospheric flows is illustrated in Fig. 2 and explained in the following:

The subatomic dynamics of quantum systems such as electron, photon, etc., are described by quantum mechanics in terms of a group of waves, i.e. a wavetrain that can be built up of a large number of sine waves of slightly differing frequency. Where the waves together produce an amplitude ψ, this region advances with a group velocity that can represent the velocity of a particle whose position is represented by the region of amplitude ψ. The

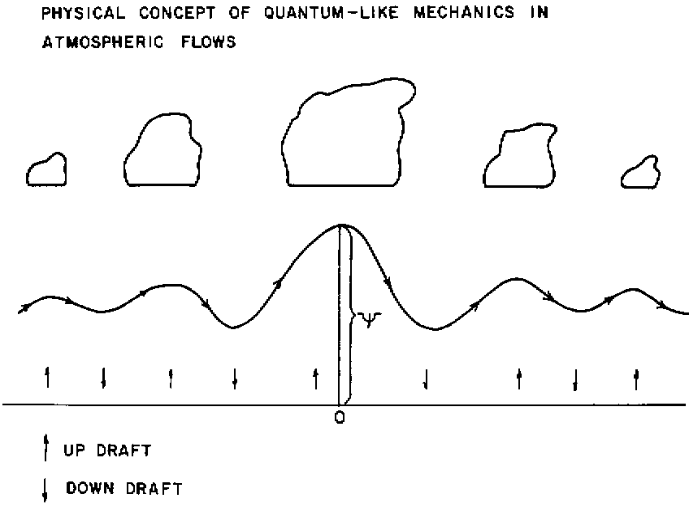

Fig. 2. Physical concept of quantum-like mechanics in atmospheric flows.

probability of finding a particle at the coordinate where eddy amplitude ψ is evaluated is then found to be proportional to ψ^2.

Atmospheric flows associated with cloud formation may also be considered to be due to wavetrains such as that shown in Fig. 2 with alternating regions of updrafts and downdrafts. The square of the eddy amplitude, i.e. ψ^2 represents eddy kinetic energy and is proportional to the intensity of the weather system at the location where ψ is measured. The passage of the wavetrain over location O at the surface will be associated with progressively increasing cloud cover followed by progressive dissipation. The signature of the passage of the wavetrain will be recorded in the meteorological parameters at O. Power spectral analysis of such meteorological time series data will enable resolution of component eddies in the wavetrain.

The conventional power spectrum plotted as the variance versus the frequency in log-log scale will now represent the eddy probability density on logarithmic scale versus the standard deviation of the eddy fluctuations on linear scale since the logarithm of the eddy wavelength represents the standard deviation, i.e. the r.m.s. value of eddy fluctuations (Eq.2). The r.m.s. value of the eddy fluctuations can be represented in terms of statistical normal distribution as follows. A normalized standard deviation $t=0$ corresponds to cumulative percentage probability density equal to 50 for the mean value of the distribution. Since the logarithm of wavelength represents the r.m.s. value of eddy fluctuations the normalized standard deviation t is defined for the eddy energy as $t=(\log L/\log T_{50})-1$ where L is the period in years and T_{50} is the period up to which the cumulative percentage contribution to total variance is equal to 50 and $t=0$. Log T_{50} also represents the mean value for the r.m.s. eddy fluctuations and is consistent with the concept of the mean level represented by r.m.s. eddy fluctuations.

In the following section it is shown that continuous periodogram analyses of rainfall time series over India exhibit the signatures of quantum-like mechanics, namely, the cumulative percentage contribution to total variance, computed starting from the high frequency end of the spectrum, follows the cumulative normal distribution.

III. DATA AND ANALYSIS

Indian region summer monsoon (June-September) rainfall for 29 meteorological sub-divisions for 115 years (1871-1985) (Parthasarathy et al., 1987) was used for this study.

The broadband power spectrum of the rainfall time series can be computed accurately by an elementary but very powerful method of analysis developed by Jenkinson (1977) which provides a quasi–continuous form of the classical periodogram allowing systematic allocation of the total variance and degrees of freedom of the data series to logarithmically spaced elements of the frequency range (0.5, 0). The periodogram is constructed for a fixed set of 10000(m) periodicities which increase geometrically as $L_m = 2 \exp(Cm)$ where $C = .001$ and $m = 0,1,2 \cdots m$. The data series Y_t for the N data points was used. The periodogram estimates the set of $A_m \cos(2\pi \nu_m t - \varphi_m)$ where A_m, ν_m and φ_m denote respectively the amplitude, frequency and phase angle for the mth periodicity. The cumulative percentage contribution to total variance was computed starting from the high frequency side of the spectrum. The period T_{50} at which 50% contribution to total variance occurs is taken as reference and the normalized standard deviation t_m values are computed as:

$$t_m = (\log L_m / \log T_{50}) - 1.$$

The cumulative percentage contribution to total variance and the corresponding t values are plotted as continuous lines in Fig. 3 for the rainfall. The cumulative normal probability density distribution corresponding to the normalized standard deviation t values is shown as crosses in Fig. 3. It is seen that the cumulative percentage contribution to total variance closely follows the cumulative normal probability density distribution. The "goodness of fit" was tested using the standard statistical chi–square test (Spiegel, 1961). The short horizontal lines in the lower part of Fig. 3 indicate the lower limit above which the fit is good at 95% confidence level.

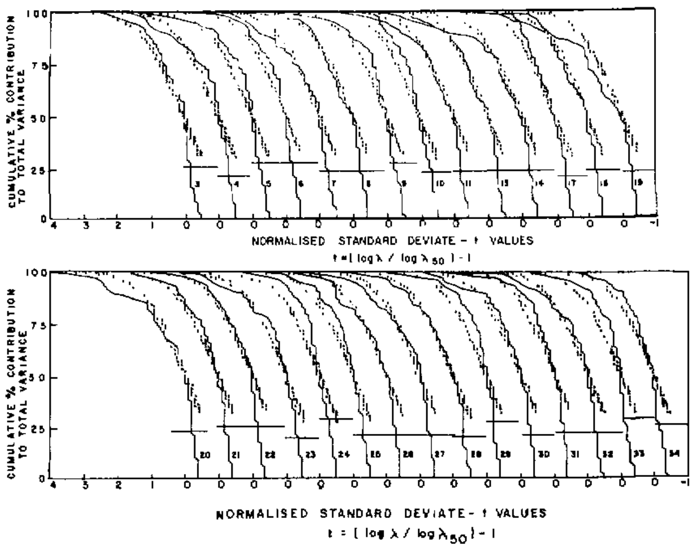

Fig. 3. Periodograms of 115 years (1871–1985) summer monsoon (June–September) rainfall for 29 meteorological sub–divisions (numbers ranging from 3 to 34) over the Indian region, plotted with progressive shift in zero by one division on the t–axis. The meteorological sub–division numbers are indicated in the lower part of the figure. The continuous line is the periodogram while the crosses refer to the corresponding cumulative normal probability density distributions. The horizontal lines on the lower part of the figure indicate the lower limit above which the periodogram is the same as the cumulative normal probability density distribution as determined by the chi square test at 95% level significance.

It is seen from Fig. 3 that the atmospheric eddy energy spectrum derived from rainfall time series for India exhibits universal characteristics of the statistical normal distribution.

IV. DISCUSSION AND CONCLUSION

It is shown that the atmospheric eddy spectrum derived from rainfall time series over India is the same as the normal probability density distribution. The normal probability distribution follows the inverse power law form t^{-B} where B, the exponent approaches 1 for small values of t. It is therefore consistent that the atmospheric eddy energy spectrum follows t^{-B} power law which is identified as the temporal signature of deterministic chaos or self–organized criticality in atmospheric flows (Mary Selvam, 1990; Mary Selvam et al., 1992). The model predictions are confirmed by determining the atmospheric eddy energy spectrum using continuous periodogram analysis technique for 115 years (1871–1985) summer monsoon (June–September) rainfall time series for 29 meteorological sub–divisions over India. The important results of the present study are as follows. Temporal (years) fluctuations in rainfall contribute to form a self–organized unique pattern, namely, that of the statistical normal distribution with the square of the eddy amplitude representing the normal probability density corresponding to the normalized standard deviation t equal to $[(\log L / \log T_{50}) - 1]$ where L is the period length in years and T_{50} the period up to which the cumulative percentage contribution to total variance is equal to 50 and $t = 0$. Quantification of the non–linear variability of atmospheric flows in terms of the unique universal characteristics of the statistical normal distribution implies predictability of the total pattern of fluctuations.

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REFERENCES

Barnett T.P. (1991), The interaction of multiple time scales in the tropical climate system, *J. Climate*, **4:** 269–285.

Bak P, Tang C and Wiesenfeld K. (1988), Self–organized criticality, *Phys. Rev.* **A38:** 364–374.

Jenkinson A.F. (1977), *A powerful elementary method of spectral analysis for use with monthly, seasonal or annual meteorological time series*, (U.K. Meteorol. Office, London) Met 0–13 Branch Memorandum No. 57, 1–23.

Lamb H.H. (1972), *Climate: Present, Past, Future, Vol. I Fundamentals and Climate Now*, (London Methuen and Co. Ltd) pp 613.

Lorenz E.N. (1990), Can chaos and intrasitivity lead to interannual variability? *Tellus*, **42A:** 378–389.

Lovjoy S and Schertzer D. (1986), Scale invariance, symmetries, fractals and stochastic simulations of atmospheric phenomena, *Bull. Am. Meteorol. Soc.*, **67:** 21–32.

Maddox J. (1988), Licence to slang Copenhagen? *Nature*, 332–581.

Mary Selvam A. (1990), Deterministic chaos, fractals and quantum–like mechanics in atmospheric flows, *Can J. Phys.*, **68:** 831–841.

Mary Selvam A, Pethkar J. S. and Kulkarni M.K. (1992), Signature of a universal spectrum for atmospheric interannual variability in rainfall time series over the Indian region: *Int' l. J. Climatol*, **12:** 137–152.

Oona Y and Puri S. (1988), Study of phase separation dynamics by use of cell dynamical systems, I. Modelling: *Phys. Rev.*, **A38:** 434–453.

Parthasarathy B., Sontakke N.A., Munot A.A. and Kothawale D.R. (1987), Droughts / floods in the summer monsoon season over different meteorological sub– divisions of India for the period 1871–1984, *J. Climatology*, **7:** 57–70.

Philander S.G. (1990), *El Nino, La Nina and the Southern Oscillation* (NY Academic Press) International Geophysical Series 46, pp 291.

Spiegel M.R. (1961), *Statistics* (New York McGraw–Hill), pp 359.

Tsonis A.A. and Elsner J. B. (1990), Multiple attractors, fractal basins and long term climate dynamics: *Beitr. Phys. Atmosph.*, **63:** 171–176.

Townsend A.A. (1956), *The structure of Turbulent Shear Flow* (U.K. Cambridge University Press), pp 130.